Calculation Method for Compressible Turbulent Boundary Flows with Heat Transfer

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Nomenclature

c_f = local skin-friction coefficient
F_1, F_2 = initial distribution functions, see Eq. (6)
h = specific enthalpy
H = total enthalpy, $h+u^2/2$
k = thermal conductivity
l = mixing length
M = Mach number
Pr = Prandtl number
R = gas constant in ideal gas equation of state
Re_x = Reynolds number, $\rho_e u_e x/\mu_e$
Re_θ = Reynolds number, $\rho_e u_e \theta/\mu_e$
St = Stanton number, $k(\partial T/\partial y)_w/\rho_e u_e(H_{aw}-H_w)$
T = absolute temperature
T_t = total or stagnation temperature
u = x component of time mean velocity
v = y component of time mean velocity
x = distance along surface
y = distance normal to wall
y^+ = dimensionless distance, $\rho_w y(\tau_w/\rho_w)^{1/2}/\mu_w$
δ = boundary-layer thickness
θ = momentum thickness
μ = dynamic viscosity
ρ = time mean density
τ = total shear stress

Subscripts

aw = adiabatic wall
e = evaluated at outer edge of boundary layer
w = evaluated at wall
T = turbulent flow

Superscripts

$(\ )'$ = fluctuating quantities
$\overline{(\ )}$ = time mean quantity

Theme

AN explicit, noniterative finite-difference calculation procedure has been developed for the two-dimensional compressible turbulent boundary layer with heat transfer. This method differs significantly from others which have been proposed recently and is, to the author's knowledge, the only explicit, noniterative finite-difference method which has been employed for the compressible turbulent boundary layer. Solutions are obtained for the velocity, temperature, and density distributions in the boundary layer from which the skin-friction coefficient and Stanton number are computed. The method can be applied to laminar as well as turbulent flows. Predicted Stanton numbers, skin-friction coefficients, velocity, and Mach number profiles are compared with experimental results for several compressible flows with Mach numbers up to 10.4 and with various degrees of wall cooling. The predictions are also compared with the theory of Van Driest[1] and the Spalding-Chi correlation.[2,3] The general level of agreement between the predictions and the compressible turbulent flow results from seven separate experimental investigations was generally good. Over the range of experiments considered, the predictions agreed more closely with the theory of Van Driest from the Spalding-Chi correlation.

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Index category: Boundary Layers and Convective Heat Transfer—Turbulent.

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Content

Neglecting normal stress terms, the equations governing the two-dimensional compressible turbulent boundary layer can be written as

Momentum:

$$\rho u\, \partial u/\partial x + (\rho v + \overline{\rho' v'})\, \partial u/\partial y = \rho_e u_e\, du_e/dx + (\partial/\partial y)(\mu\, \partial u/\partial y - \rho' \overline{u' v'}) \tag{1}$$

Energy:

$$\rho u\, \partial H/\partial x + (\rho v + \overline{\rho' v'})\, \partial H/\partial y = (\partial/\partial y)\left[(k/c_p)\, \partial h/\partial y - \rho \overline{v' h'} + u(\mu\, \partial u/\partial y - \rho \overline{u' v'})\right] \tag{2}$$

Continuity:

$$(\partial/\partial x)(\rho u) + (\partial/\partial y)(\rho v + \overline{\rho' v'}) = 0 \tag{3}$$

State:

$$p/\rho = RT \tag{4}$$

Appropriate boundary conditions are

$$u(x,0)=0,\quad v(x,0)=0,\quad H(x,0)=H_w \quad \text{or} \quad (\partial H/\partial y)(x,0) = (\partial H/\partial y)_w,\quad \lim_{y\to\infty} u(x,y) = u_e(x),\quad \lim_{y\to\infty} H(x,y) = H_e(x) \tag{5}$$

In addition, initial values must be provided for the axial component of velocity and the enthalpy:

$$u(x_o, y) = F_1(y), \quad H(x_o, y) = F_2(y) \tag{6}$$

and $u_e(x)$ must be specified for pressure gradient flows.

Equations (1–4) cannot be solved without additional information being provided or a relationship between the quantities $-\rho\overline{u'v'}$ and $-\rho\overline{v'h'}$ and the velocity and enthalpy distributions assumed. Although the numerical method described in the full paper can be used with various models for the turbulent transport, good results have been obtained to date with the following model.

Using Prandtl's mixing length concept, it is assumed that

$$-\rho\overline{u'v'} = \rho l^2 |\partial u/\partial y|\, \partial u/\partial y \tag{7}$$

where $\rho l^2 |\partial u/\partial y|$ will be identified as μ_T, a turbulent or "eddy" viscosity. Thus, the total or effective shearing stress can be described by

$$\tau = (\mu + \mu_T)\, \partial u/\partial y \tag{8}$$

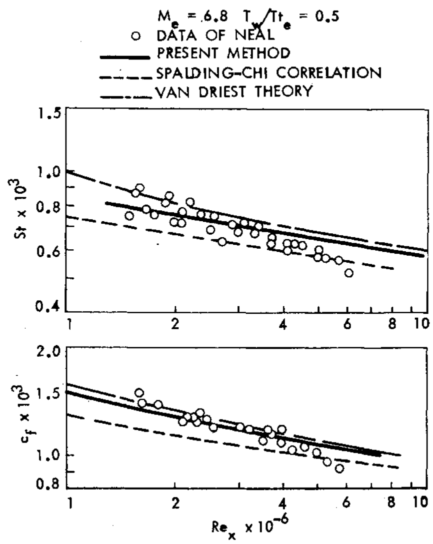

Fig. 1 Comparison of calculated and experimental results for the boundary layer measured by Neal,[6] $M_e = 6.8$.

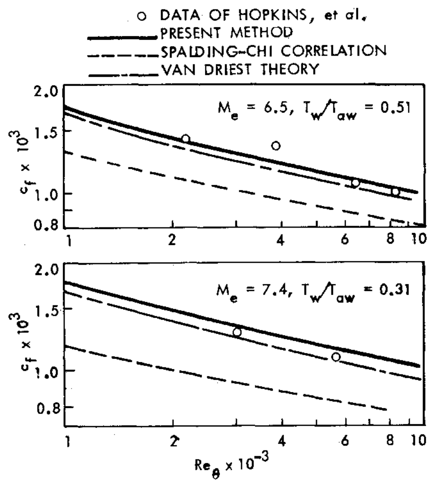

Fig. 2 Comparison of calculated and experimental results for the boundary layer measured by Hopkins et al.[7] $M_e = 6.5$, $T_w/T_{aw} = 0.5$, and $M_e = 7.4$, $T_w/T_{aw} = 0.31$.

It is assumed that the turbulent diffusivities for heat and momentum are related in a manner analogous to laminar flows resulting in

$$k_T = c_p \mu_T / Pr_T \tag{9}$$

where k_T is the turbulent or "eddy" conductivity and Pr_T is the turbulent Prandtl number which was set equal to a constant value of 0.9 for all predictions of this paper. The turbulent conductivity concept allows

$$-\rho\overline{v'h'} = (\mu_T/Pr_T)\,\partial h/\partial y = (\mu_T/Pr_T)\,\partial H/\partial y - (\mu_T/Pr_T) u\,\partial u/\partial y \tag{10}$$

It is assumed that the mixing length l is given by

$$l/\delta = 0.42(1-e^{-y^+/26})y/\delta, \quad y/\delta \leqq 0.089/0.42(1-e^{-y^+/26}) \tag{11}$$

$$l/\delta = 0.089, \quad y/\delta \geqq 0.089/0.42(1-e^{-y^+/26}) \tag{12}$$

Using this assumed mixing length distribution and the results of Eqs. (7–10), the turbulent transport quantities $-\rho\overline{u'v'}$ and $-\rho\overline{v'h'}$ can be expressed in terms of the mean velocity and enthalpy fields. The resulting system of partial differential equations are solved by a stable explicit finite-difference procedure which combines the unrestricted stability feature of implicit methods with the computational simplicity of explicit methods. A description of the finite-difference formulation is given in the full paper. Additional details of the development and some results for incompressible flows and adiabatic compressible flows are given in Refs. 4 and 5.

Comparisons of the predictions with 12 sets of experimental data from seven different experimental programs are shown in the full paper. Most of the cases reported required about one minute of computation time and none more than two minutes on the IBM 360/65. Figures 1 and 2 are representative of the comparisons for skin-friction coefficient and Stanton number. The experimental skin friction was obtained by the floating element technique. Figure 1 compares the predicted local Stanton number and skin-friction coefficient with the measurements of Neal[6] at $M_e = 6.8$ and $T_w/T_{t_e} = 0.5$. Figure 2 compares the predictions and the measurements of Hopkins et al.[7] for $M_e = 6.5$, $T_w/T_{aw} = 0.51$ and $M_e = 7.4$, $T_w/T_{aw} = 0.31$. The general trends observed in all comparisons made to date indicate close agreement between predictions of the present method and that of the Van Driest[1] theory. The agreement between both theories and the experimental data was generally good. The Spalding-Chi[2] correlation underpredicted the experimental data by 15–30% for the higher Mach numbers ($M_e \geqq 5$).

This work demonstrates that a stable explicit finite-difference scheme can be developed to solve the energy equation in partial differential form simultaneously with the momentum and continuity equations for the compressible turbulent boundary layer with heat transfer. The method is more direct than most proposed recently since no transformations (other than a simple nondimensionalization) are employed and no iterative procedures are required. As far as can be determined from statements in the literature, the method appears to be at least as fast as any other differential method.

Further, the relatively simple mixing length model, which works well for constant property flows and modeling for the turbulent transport of heat proposed here, appears to properly account for the effects of property variations for most purposes in compressible turbulent flows with heat transfer at least up through about $M_e = 7.4$ and to levels of wall cooling corresponding to $T_w/T_{aw} = 0.3$.

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